(1) $F(x) = \frac{\sqrt{x} - 3}{x - 9}$ as x approaches 9 from left and right.

$x < 9$	$F(x)$	$x > 9$	$f(x)$
8.5	0.45	9.5	0.16441
8.9	0.167	9.1	0.16620
8.99	0.1667	9.01	0.16620
8.999	0.16667	9.001	0.166662
8.9999	0.166667	9.0001	0.1666662

As x approaches 9, the value of the function seems to approach 0.166666 --- and so we guess that value of the function.

(b) Use the table to estimate $\lim_{x \to 9} f(x)$

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} = \frac{1}{6}$$

$\sqrt{x} \times 1$

1(c) Using the appropriate factorization, find an exact value for $\lim_{x \to 9} f(x)$

Given $f(x) = \frac{\sqrt{x} - 3}{x - 9}$

$$= \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$$

$$= \lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9} \cdot \frac{\sqrt{x} + 3}{\sqrt{x} + 3}$$

$$\lim_{x \to 9} \frac{x + 3\sqrt{x} - 3\sqrt{x} - 9}{x\sqrt{x} + 3x - 9\sqrt{x} - 27}.$$

$$= \lim_{x \to 9} \frac{x - 9}{x\sqrt{x} + 3x - 9\sqrt{x} - 27}.$$

$$= \lim_{x \to 9} \frac{\cancel{(x-9)}\ 1}{\cancel{(x-9)}(\sqrt{x} + 3)}$$

$$= \lim_{x \to 9} \frac{1}{\sqrt{x} + 3}$$

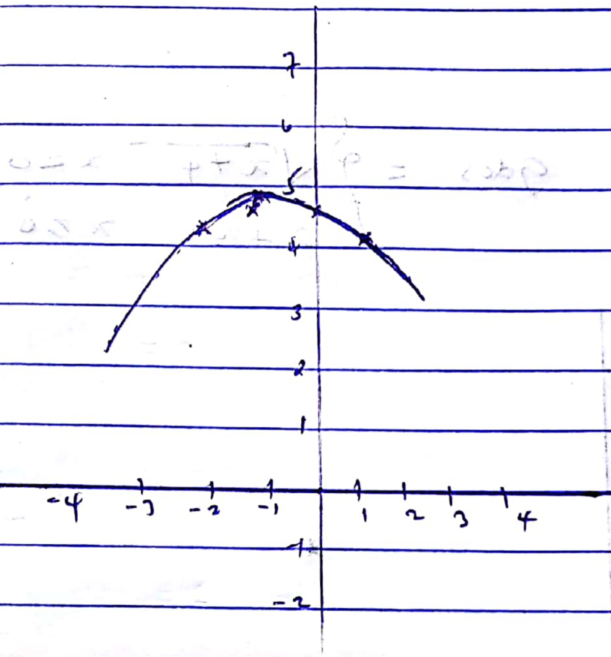
$$= \lim_{x \to 9} \frac{1}{\sqrt{x} + 3}$$

$$= \frac{1}{\sqrt{9} + 3}$$

$$= \frac{1}{6}$$

2 Sketch the graph

$f(x) = \frac{2}{x-4} + 5$



$\lim_{x \to 4} \frac{2}{x-4} + 5$

$= \lim_{x \to 4} \frac{2}{x-4} + \frac{5}{1} \quad \frac{2+5(x-4)}{x-4}$

$= \lim_{x \to 4} \frac{2+5x-4}{x-4}$

$= \lim_{x \to 4} \frac{2+5x-4}{x-4} \cdot \frac{2+5x-4}{2+5x-4}$

$= \lim_{x \to 4} \frac{5x-2}{x-4} \frac{(5x+2)}{(5x+2)}$

$\lim_{x \to 4} \frac{5x^2 + \cancel{10x} - \cancel{10x} - 4}{(x-4)(5x+2)}$

$\lim_{x \to 4} \frac{5x^2 - 4}{5x^2 + 2x - 20x - 8}$

$\lim_{x \to 4} \frac{5x^2 - 4}{5x^2 + 18x - 8}$

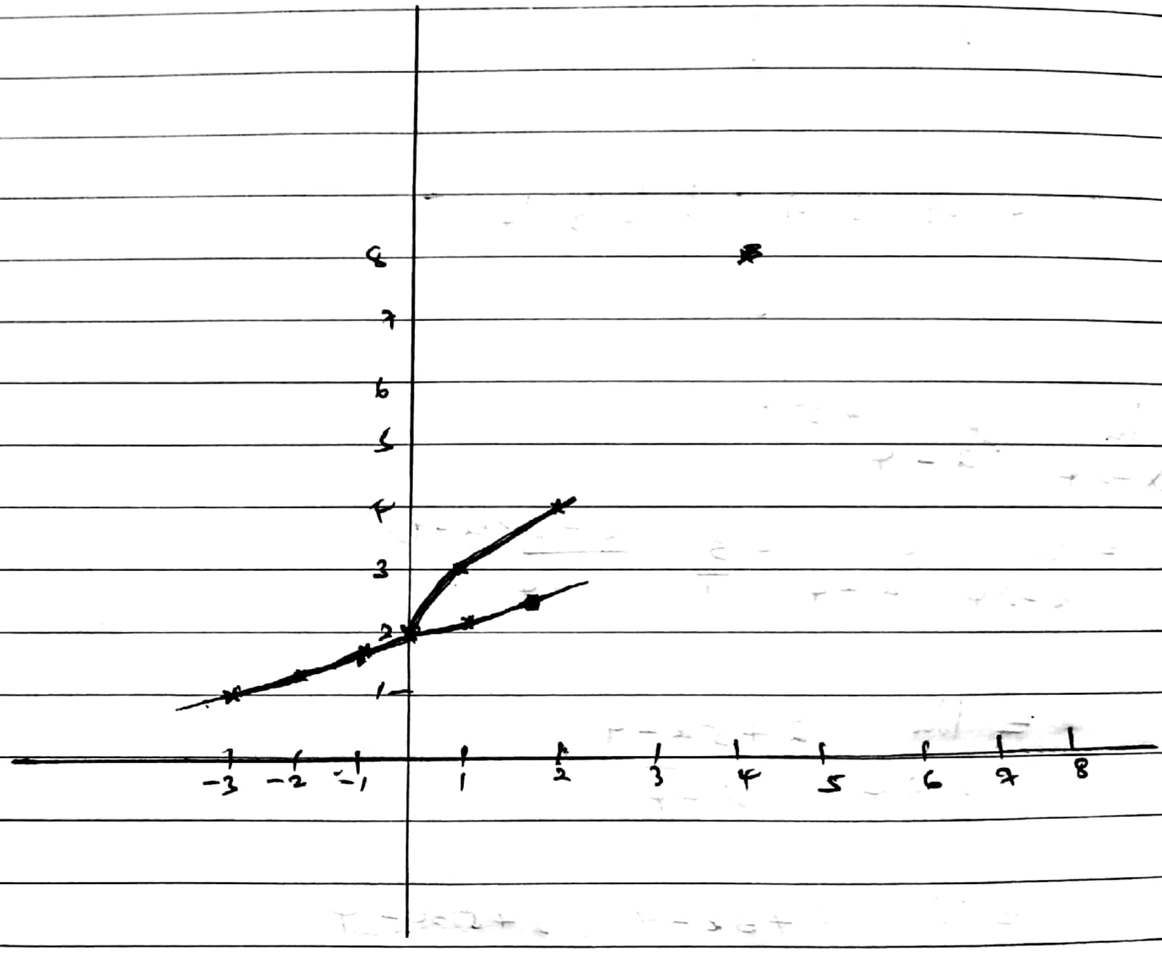
$= \frac{-4}{-8}$

$= \frac{1}{2}$

$\lim_{x \to 4} k(x)$ exist

② consider function $g(x) = \begin{cases} \sqrt{x+4} & x \geq 0 \\ 2+x & x \geq 0 \end{cases}$

(a)



(b) $\lim_{x \to 0^-} g(x) = 2$

(c) $\lim_{x \to 0^+} g(x) = 2$

(d) $\lim_{x \to 0} g(x) = 2$

(i) Is g continous at 0? g(x) is continous at every point of the domain of g(x).

(4) Consider the function $h(x)$, graphed below.

Find the following limits. If the limits does not exist explain why.

(i) $\lim_{x \to -1^-} h(x)$

$\lim_{x \to -1^-} h(x) = 1$

(ii) $\lim_{x \to -1^+} h(x) = 0$

(iii) $\lim_{x \to -1} h(x)$ Doe not exist. The right-hand and left hand limits are not equal.

(iv) $\lim_{x \to 0^-} h(x) = 1$

(v) $\lim_{x \to 0^+} h(x) = 1$

(vi) $\lim_{x \to 0} h(x) = 1$

(vii) $\lim_{x \to 1^-} h(x) = 0$

(viii) $\lim_{x \to 1^+} h(x) = 0$

(ix) $\lim_{x \to 1} h(x)$ Does not exist. The function is continous at the point $x = 1$.

(x) $\lim_{x \to -2} h(x)$ Does not exist. The function is not defined to the right of $x = -2$

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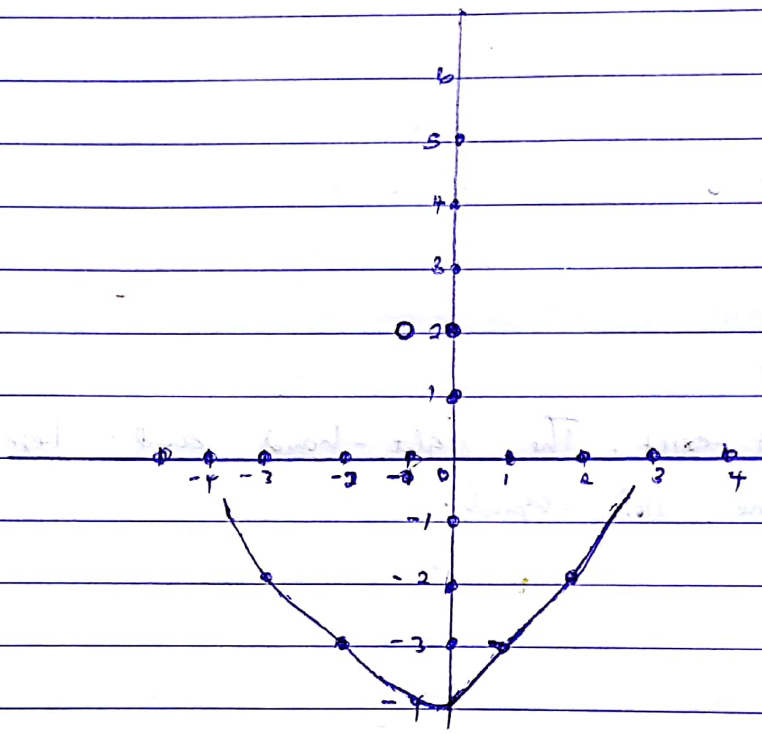
$4-6-4$
$-2-6$ $-\frac{6}{3}$ 3 $\frac{3}{-1}$
-2
$4+3-4$

(c) let $F(x) = \begin{cases} \dfrac{x^2-3x-4}{x+1} & x \neq -1 \\ 2 & x = -1 \end{cases}$

(a) Sketch the graph of F



(b) Is F continuous at $x = -1$? Explain why or not Using definition of Continuity. The function $F(x) = \begin{cases} \dfrac{x^2-3x-4}{x+1} & x \neq -1 \\ 2 & x = -1 \end{cases}$ is continuous function because it is continuous at every point of its domain. It has a point of discontinuity at at $x = -1$, however because it is not defined there; that is, it is discontinuous on any interval containing $x = -1$.

(b) (a) What is the initial number of residents.

The initial number of residents is given when t is at initial point that is $t=0$

Initial number of residents $= 0.1t^2 + 2t + 5$. replace t with 0

Initial number of residents $= 0.1(0^2) + (2 \times 0) + 5 = 5$

$N(t)$ initial $= \underline{\underline{500}}$

(b) Average rate of change of N over [1 4]

$$= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$f(1) = 0.1 + 2 + 5$.

$f(1) = 7.1$

$f(4) = (0.1 \times 4) + (2 \times 4) + 5$

$f(4) = 0.4 + 8 + 5$

$f(4) = 13.4$

$$= \frac{\Delta y}{\Delta x} = \frac{f(4) - f(1)}{4 - 1}$$

$$= \frac{13.4 - 7.1}{3}$$

$$= \frac{6.3}{3} = 2.1$$

Average rate of change of $N = \underline{\underline{2.1}}$

(c) Explain the number in (b) represents

It represents the rate at which Population grows over a given period of time (1 year)

7 For $f(x) = 5x^2 + 3x - 2$

(a) Find the simplified form of the difference quotient.

$\Delta y = f(x_2) - f(x_1)$

$\Delta x = x_2 - x_1$

$$f(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{[5(a+h)^2 + 3(a+h) - 2] - [5a^2 + 3a - 2]}{h}$$

$$= \lim_{h \to 0} \frac{5[a^2 + 2ah + h^2] + 3a + 3h - 2 - 5a^2 - 3a + 2]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{5a^2} + 10ah + 5h^2 + \cancel{3a} + 3h - \cancel{2} - \cancel{5a^2} - \cancel{3a} + \cancel{2}}{h}$$

$$= \lim_{h \to 0} \frac{10ah + 5h^2 + 3h}{h}$$

$$= \lim_{h \to 0} 10a + 5h + 3$$

$$f'(a) = \lim_{h \to 0} 10a + 5h + 3$$

$$f'(a) = 10a + 3$$

(b) $f'(-1)$

$$f'(-1) = 10(-1) + 3$$

$$f'(-1) = -7$$

Equation of the tangent line at $x = -1$

at $x = -1$

$y = 5x^2 + 3x - 2$

$y = 5 - 3 - 2$

$y = 0$

$y - 0 = -7(x + 1)$

$y = -7x - 7$

8

(a) For $f(x) = \frac{3}{5-2x}$

(a) Find the simplified form of the difference quotient

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{\frac{1}{5-2(a+h)} - \frac{1}{5-2(a)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{5-2a+2h} - \frac{1}{5-2a}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(5-2a) - (5-2a+2h)}{(5+2a)(5-2a+2h)}}{h}$$

$$\lim_{h \to 0} \frac{5-2a-5+2a+2h}{h(5-2a)\ 5-2a+2h}$$

$$\lim_{h \to 0} \frac{5-2a-5 \overset{0}{\cancel{+2a}}}{h(5-2a)(5-2a-h)} + \frac{2h}{h(5-2a)(5-2a-h)}$$

$$= \lim_{h \to 0} \frac{2}{(5-2a)(5-2a)-h}$$

$$= \frac{2}{(5-2a)(5-2a)}$$

$$f(x) = \frac{2}{5-2a}$$

(b) $f(1)$

$$f'(1) = \frac{2}{5-2a}.$$

$$f'(1) = 2/3$$

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(c) Equation of the tangent line at $X = 1$

$y = \frac{3}{5-2x}$ but $x = 1$

$y = \frac{3}{5-2}$

$= 3/3$

$y = 1$

Equation of the tangent $=$ $y - 1 = 2/3\,(x-1)$

$= y - 1 = 2/3\,x - 1$

$y = 2/3\,x$

8

(9) (a) Given the x-values at which h is not continuous
Explain: The function h(x) is not continuous at point $x = c$ if ~~and only~~ if it does not meet the following conditions
(i) c lies in the domain of h
(ii) h has a limit as $x \to c$
(iii) The limit equals the function value.

(b) Given the x-values at which h is not differentiable
Explain: A function h(x) is not differentiable on an open interval if it has no derivative at each point of the interval. It is not differentiable on closed interval [a b] if it is not differentiable on the interior (a b) it has no limits